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Numerical Methods - MA 207
Difference Equations

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1. Convert the followind difference equations into recurrence relations (in the subscript notations).
   (a) $\Delta^3 y_x - 3\Delta^2 y_x + 2\Delta y_x + y_x = 0$
   (b) $\Delta^2 u_x - \Delta u_x + 3u_x = x^2$.
2. Find order and degree of the following difference equations.
   (a) $\Delta y_n + y_n = n$
   (b) $\Delta^2 u_x - 4\Delta u_x + 4u_x = 3^x$
   (c) $4y_{n+3}^2 - 2y_n y_{n+1} + y_n^2 y_{n+1}^4 = 0$.
3. Find order and degree of the difference equation
$$\Delta^3 y_n - 3\Delta^2 y_n + 2\Delta y_n + y_n = \cos \pi n.$$
4. Verify the following:
   (a) $y_x = A\,2^x + B\,3^x$ is a solution of $y_{x+2} - 5y_{x+1} + 6y_x = 0$.
   (b) $y_n = 1 - \frac{2}{n}$ is a solution of the difference equation
$$(n+1)y_{n+1} + ny_n = 2n - 3.$$
   (c) $y_x = 2^x(c_1 + c_2 x)$ is a solution of
$$y_{x+3} - 4y_{x+1} + 4y_x = 0, \quad x = 0, 1, 2, \ldots.$$
   Find the particular solution when $y_0 = 1$ and $y_1 = 6$.
5. Find the difference equation satisfied by $y = ax^2 - bx$.
6. Form the difference equation of the lowest possible order by eliminating the constants A and B, from
$$y_n = Aa^n + Bb^n$$
   where $a \neq b$.
7. Form the difference equation by eliminating the constant 'a' from $y = a3^n$.
8. Given $f(x) = c3^x + x3^{x-1}$, find the corresponding difference equation.
9. Given
$$u_x = c_1 2^x + c_2 3^x + \frac{1}{2}$$
   find the corresponding difference equation.

10. Form the difference equations corresponding to the family of curves.

  (a) $y = ax + bx^2$
  (b) $y_n = a \sin n\theta + b \cos n\theta$.

11. Show that n circles drawn in a plane so that each circle intersects all the others and no three circles meet in a point, divide the plane into $(n^2 - n + 2)$ parts.

12. Show that n straight lines, no two of which are parallel and no three of which meet in a point, divide the plane into $\frac{1}{2}(n^2 + n + 2)$ parts.

13. Solve the following difference equations.

  (a) $u_{n+3} - 2u_{n+2} - 5y_{n+1} + 6u_n = 0$
  (b) $u_{n+2} - 2u_{n+1} + u_n = 0$
  (c) $y_{n+1} - 2y_n \cos \alpha + y_{n-1} = 0$
  (d) $(E^2 + E + 1)y_n = 0$.

14. The integers $0, 1, 1, 2, 3, 5, 8, 13, \ldots$ are said to form a **Fibonacci sequence**. Form the difference equation (recurrence relation) and solve it.

15. Solve $(E^3 - 5E^2 + 8E - 4)y_n = 0$ given that $y_0 = 3,\ y_1 = 2,\ y_4 = 22$.

16. Solve $u_{n+2} + u_n = 5(2)^n$ given $u_0 = 1,\ u_1 = 0$.

17. If $y_0 = 2$, solve the difference equation

$$y_{x+1} + 3y_x = 0, \quad x = 0, 1, 2, \ldots.$$

18. Solve $y_{x+1} - y_x = (x^2 - 2x)2^x$.

19. Solve the following difference equations:

  (a) $y_{x+2} + y_{x+1} + y_x = x^2 + x + 1$
  (b) $y_{x+2} - 4y_x = 2^x$
  (c) $y_{k+2} - 2y_{k+1} + 5y_k = 4(3)^k - 10(7)^k$
  (d) $y_{k+2} - 4y_{k+1} + 4y_k = 3(2)^k + 5(4)^k$.

20. Solve the following difference equations:

  (a) $y_{n+2} - 2\cos \alpha y_{n+1} + y_n = \cos \alpha n$
  (b) $y_{n+2} - 2y_{n+1} + y_n = n^2\, 2^n$.

21. Solve the simultaneous difference equations

$$u_{x+1} + v_x - 3u_x = x, \quad \text{and} \quad 3u_x + v_{x+1} - 5v_x = 4^x$$

subject to the conditions $u_1 = 2,\ v_1 = 0$.

22. Solve the simultaneous difference equations

$$y_{n+1} - y_n + 2z_{n+1} = 0, \quad \text{and} \quad z_{n+1} - z_n = 2y_n = 2^n.$$

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